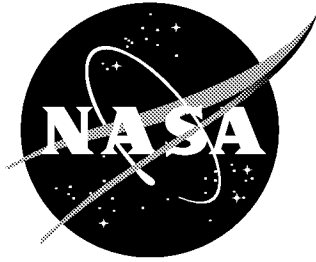


NASA/TM-1999-208974



# Comparing Parameter Estimation Techniques for an Electrical Power Transformer Oil Temperature Prediction Model

*A. Terry Morris*  
*Langley Research Center, Hampton, Virginia*

---

January 1999

## The NASA STI Program Office ... in Profile

Since its founding, NASA has been dedicated to the advancement of aeronautics and space science. The NASA Scientific and Technical Information (STI) Program Office plays a key part in helping NASA maintain this important role.

The NASA STI Program Office is operated by Langley Research Center, the lead center for NASA's scientific and technical information. The NASA STI Program Office provides access to the NASA STI Database, the largest collection of aeronautical and space science STI in the world. The Program Office is also NASA's institutional mechanism for disseminating the results of its research and development activities. These results are published by NASA in the NASA STI Report Series, which includes the following report types:

- **TECHNICAL PUBLICATION.** Reports of completed research or a major significant phase of research that present the results of NASA programs and include extensive data or theoretical analysis. Includes compilations of significant scientific and technical data and information deemed to be of continuing reference value. NASA counterpart of peer-reviewed formal professional papers, but having less stringent limitations on manuscript length and extent of graphic presentations.
- **TECHNICAL MEMORANDUM.** Scientific and technical findings that are preliminary or of specialized interest, e.g., quick release reports, working papers, and bibliographies that contain minimal annotation. Does not contain extensive analysis.
- **CONTRACTOR REPORT.** Scientific and technical findings by NASA-sponsored contractors and grantees.

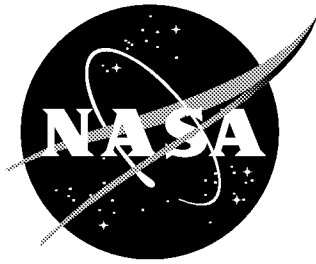
- **CONFERENCE PUBLICATION.** Collected papers from scientific and technical conferences, symposia, seminars, or other meetings sponsored or co-sponsored by NASA.
- **SPECIAL PUBLICATION.** Scientific, technical, or historical information from NASA programs, projects, and missions, often concerned with subjects having substantial public interest.
- **TECHNICAL TRANSLATION.** English-language translations of foreign scientific and technical material pertinent to NASA's mission.

Specialized services that complement the STI Program Office's diverse offerings include creating custom thesauri, building customized databases, organizing and publishing research results ... even providing videos.

For more information about the NASA STI Program Office, see the following:

- Access the NASA STI Program Home Page at <http://www.sti.nasa.gov>
- E-mail your question via the Internet to [help@sti.nasa.gov](mailto:help@sti.nasa.gov)
- Fax your question to the NASA STI Help Desk at (301) 621-0134
- Phone the NASA STI Help Desk at (301) 621-0390
- Write to:  
NASA STI Help Desk  
NASA Center for AeroSpace Information  
7121 Standard Drive  
Hanover, MD 21076-1320

NASA/TM-1999-208974



# Comparing Parameter Estimation Techniques for an Electrical Power Transformer Oil Temperature Prediction Model

*A. Terry Morris*  
*Langley Research Center, Hampton, Virginia*

National Aeronautics and  
Space Administration

Langley Research Center  
Hampton, Virginia 23681-2199

---

January 1999

---

Available from:

NASA Center for AeroSpace Information (CASI)  
7121 Standard Drive  
Hanover, MD 21076-1320  
(301) 621-0390

National Technical Information Service (NTIS)  
5285 Port Royal Road  
Springfield, VA 22161-2171  
(703) 605-6000

# Comparing Parameter Estimation Techniques for an Electrical Power Transformer Oil Temperature Prediction Model

A. Terry Morris  
NASA Langley Research Center

*Abstract* - This paper examines various sources of error in MIT's improved top oil temperature rise over ambient temperature model and estimation process. The sources of error are the current parameter estimation technique, quantization noise, and post-processing of the transformer data. Results from this paper will show that an output error parameter estimation technique should be selected to replace the current least squares estimation technique. The output error technique obtained accurate predictions of transformer behavior, revealed the best error covariance, obtained consistent parameter estimates, and provided for valid and sensible parameters. This paper will also show that the output error technique should be used to minimize errors attributed to post-processing (decimation) of the transformer data. Models used in this paper are validated using data from a large transformer in service.

## 1 Introduction

The failure of large power transformers is a considerable concern for electric utility companies. Encased in tanks of flammable and environmentally hazardous fluids, large power transformers that fail present considerable danger to people, property and the local environment. Large power transformers are also costly devices. The capital costs alone associated with repairing or replacing a large power transformer that has suffered catastrophic failure are as much as \$1,000,000. For these reasons, utility companies have clear incentives to identify and diagnose incipient failures in in-service transformers before the failures become catastrophic.

The Massachusetts Institute of Technology (MIT) has developed an adaptive, intelligent system for on-line monitoring and diagnosis of large power transformers. This system is currently implemented on a number of power transformers, owned and operated by several utility companies in the United States. MIT's adaptive, on-line system uses a model-based monitoring approach that gives immediate indication of incipient failures in order to prevent catastrophic failures. Adaptive models, like those used in MIT's system, are advantageous in that they tune themselves to each transformer using parameter estimation. The diagnostic part of the system is a hierarchical scheme which allows both near real-time responses (to avoid catastrophic failure of the transformer), as well as long-term trends. In operation, a transformer monitor in the system takes data every five minutes. Each module in the system then compares its data with a model prediction and generates a residual error.

MIT's transformer monitoring group initially used the IEEE/ANSI top oil temperature rise over ambient temperature (TOT) model as a reasonable starting point to model the transformer's thermal behavior. After analyzing data collected from large transformers in service, the monitoring group successfully argued that the IEEE/ANSI standard TOT model does not accurately account for variations in ambient temperature [1]. A slightly modified TOT model was found that adequately incorporated ambient temperature variations. The modified model gave an improved performance over the standard model. The mean and variance of the error in predicted oil temperature were significantly smaller, compared to the standard model.

This paper provides a next step in the above process by examining various sources of errors in the improved model and estimation process and by proposing modifications to the on-line system to effectively minimize errors introduced by the estimation of parameters, the quantization noise found in the collected data, and the effects of post-processing (data sampling and constant biases). The approach in this paper is to:

- choose the best overall parameter estimation technique to calculate parameters in the transformer model while analyzing whether or not the addition of a quantization term to the transformer model improves the model's prediction; and
- examine the effects of decimation (missing data) and bias on the parameter estimates.

Results in this paper will follow a systematic process as described in the approach. The systematic process begins by first determining which parameter estimation technique produces better parameters for the improved model with no specific compensation for quantization disturbances, that is, the noise is not assumed to come from quantization effects. Second, the process will endeavor to find the best parameter estimation technique that significantly reduces the effects of quantization noise. Finally, the process will conclude by determining the effects of decimation and bias of the transformer data in order to reduce errors in the parameter estimates attributed to post processing.

Numerous parameter estimation techniques are applied to estimate the parameters in the transformer model some with and without corrections for quantization error. The techniques are: least-squares (LS), prediction error (PE), output error (OE), optimal instrumental variables (IV4), and maximum likelihood (ML) estimation.

Analysis and background of the on-line monitoring system, the original model, and the improved model will be described in section 3. Results of the process will be described in section 4. Conclusions and recommendations will follow in section 5.

## 2 Symbols and Acronyms

### Symbols

$\theta_{top}$	top oil temperature
$\theta_{amb}$	ambient temperature
$\theta_o$	top oil temperature rise over ambient temperature
$\theta_{fl}$	full load top oil temperature rise over ambient temperature
$\theta_i$	initial top oil temperature rise over ambient temperature
$T_o$	thermal time constant of the transformer
$\Delta t$	sampling period
$t$	continuous time
$k, \tau$	discrete time
$I$	load current
$I_{rated}$	rated load
$R$	ratio of load loss at rated load to no-load loss
$n$	cooling state of the transformer
$\Phi$	regressor matrix
$E\{\}$	expected value
$\mathbf{E}$	error vector
$\mathbf{Y}$	observation vector
$\hat{\theta}$	parameter vector
$K$	generic estimated parameter
$e_q[k]$	discrete quantization error
$q_n[k]$	discrete quantization disturbance
$\mu_e$	residual mean
$\sigma_e^2$	residual variance
$\hat{a}, \hat{b}$	parameter estimates

$\hat{y}$	observation estimate
$u$	input
$y$	observation
$\phi$	regressor vector
$e(t)$	additive errors

### **Acronyms**

ANSI	American National Standards Institute
IEEE	Institute of Electrical and Electronics Engineers
IV4	Instrumental Variables technique applied to the improved TOT model
IV4 (qn)	Instrumental Variables technique with compensation for quantization noise
LS	Least Squares technique
ML	Maximum Likelihood technique
MIT	Massachusetts Institute of Technology
NASA	National Aeronautics and Space Administration
OE	Output Error technique applied to the improved TOT model
OE (qn)	Output Error technique with compensation for quantization noise
PE	Prediction Error technique
QN	Quantization Noise
TOT	Top oil temperature rise over ambient temperature

## **3 Analysis and Background**

### **3.1 Adaptive, On-Line Monitoring System**

The on-line monitoring system compares each transformer's measurements (observations) with predictions obtained from simulation models. A residual signal results by computing the difference between the output of the transformer and the output of the simulation model (figure1 (a)). Transformer failures can be detected in two ways: first, by checking for large deviations in the residual signal and second, by tracking parameter trends [2]. Rapidly developing parameter trends may indicate problems with the transformer. Long-term trends in the parameter estimates provide information on natural aging. The adaptive model-based system was successfully implemented in the MIT pilot transformer facility [3,4]. The adaptive models, whose parameters are obtained using parameter estimation, are advantageous in that they have the ability to tune themselves to each transformer. The adaptation of parameters can occur in specified intervals (daily, weekly, etc.). The process generally involves the determination of parameters that best describe past data. These parameters, in turn, are used to predict transformer behavior.

### **3.2 IEEE Model**

The IEEE/ANSI C57.115 transformer top oil temperature rise over ambient temperature model [5] is based on the theory that an increase in the current of a transformer results in losses which in effect increase the overall temperature of the device. The IEEE model represents a first-order exponential response from an initial temperature state to a final temperature state:

$$\theta_o = \left( \theta_{fl} \left( \frac{\left( \frac{I}{I_{rated}} \right)^2 R + 1}{R + 1} \right)^n - \theta_i \right) \left( 1 - e^{\frac{-t}{T_o}} \right) + \theta_i \quad (1)$$

Equation (1) is the solution of the first-order differential equation

$$T_o \frac{d\theta_o}{dt} = -\theta_o + \theta_{fl} \left( \frac{\left( \frac{I}{I_{rated}} \right)^2 R + 1}{R + 1} \right)^n, \quad \theta_o(0) = \theta_i \quad (2)$$

Using

$$\frac{d\theta_o[k]}{dt} \approx \frac{\theta_o[k] - \theta_o[k-1]}{\Delta t} \quad (3)$$

as a forward Euler approximation for the time derivative, the IEEE transformer TOT model can be described by the following difference equation:

$$\theta_o[k] = \frac{T_o}{T_o + \Delta t} \theta_o[k-1] + \frac{\Delta t \theta_{fl}}{T_o + \Delta t} \left( \frac{\left( \frac{I[k]}{I_{rated}} \right)^2 R + 1}{R + 1} \right)^n \quad (4)$$

Approximations of the IEEE TOT model have been used in the MIT monitoring system with a high degree of accuracy using the forced cooling state ( $n = 1$ ). However, when applied to several large power transformers in the field, the model approximations produce results that are less than satisfactory, that is, the model predictions did not adequately represent the top oil temperature.

### 3.3 MIT's Improved Model

The transformer monitoring group at MIT when faced with unsatisfactory results of the IEEE TOT model made a slight modification to the original model. The improved TOT model adequately incorporates ambient temperature variations for the forced cooling state ( $n = 1$ ) by using the ambient temperature variable as an input. The top oil temperature error mean and variance for the improved model were found to be 0 and .4, respectively using one week of data collected from a large power transformer in service. These results were computed using the least squares technique and are extremely good as compared to a mean error of -1.51 and an error variance of 15 obtained from the IEEE model using the same data set. Results from the improved model (5) give improved performance over the IEEE model (4) and satisfy the requirements for an on-line monitoring system [1]. The equation for the improved model is

$$\begin{aligned} \theta_{top}[k] &= \frac{T_o}{T_o + \Delta t} \theta_{top}[k-1] + \frac{\Delta t}{T_o + \Delta t} \theta_{amb}[k] + \frac{\Delta t \theta_{fl} R}{(T_o + \Delta t)(R+1)} \left( \frac{I[k]}{I_{rated}} \right)^2 + \frac{\Delta t \theta_{fl}}{(T_o + \Delta t)(R+1)} \\ &= K_1 \theta_{top}[k-1] + (1 - K_1) \theta_{amb}[k] + K_2 I[k]^2 + K_3 \end{aligned} \quad (5)$$

where

$$\theta_o = \theta_{top} - \theta_{amb}, \quad K_1 = \frac{T_o}{T_o + \Delta t}, \quad K_2 = \frac{K_3 R}{I_{rated}^2}, \text{ and } K_3 = \frac{\Delta t \theta_{fl}}{(T_o + \Delta t)(R+1)}.$$



The physical model parameters ( $T_o$ ,  $\theta_{fl}$ ,  $R$ ) are computed from the estimated parameters ( $K_1$ ,  $K_2$ ,  $K_3$ ) as follows:

$$T_o = \frac{\Delta t K_1}{1 - K_1}, \quad \theta_{fl} = \frac{K_2 I_{rated}^2 + K_3}{1 - K_1}, \quad R = \frac{K_2 I_{rated}^2}{K_3} \quad (6)$$

A thorough detail of the results (including plots and tables) for the improved model are found in [1].

### 3.4 Sources of Error

There are possible sources of error in the transformer model, as described in [1]. Though the focus of [1] was on modeling, the same sources of error exist in the improved model. The first source of error may be the least-squares parameter estimation technique. The second source of error may be the quantization noise introduced by a 1°C measurement resolution in the time-varying, temperature data. The third source may be the five minute sampling period originally selected to enable the consistent estimation of parameters. The described sources of error will be addressed in the next section.

## 4 Results

This section will describe both the parameter estimation techniques used and the criteria used to select the best estimation technique. As seen in figures 1(b) and 1(c), the inputs of the on-line monitoring system are corrupted by quantization noise due to a 1°C measurement resolution. The output (not shown) is also corrupted. This may or may not be a problem for the estimation of parameters. To determine if the 1°C measurement resolution introduces a deleterious effect into the parameter estimates, two separate analyses will be conducted. In the first analysis, techniques will be compared with no apparent attention to the quantization effects, that is, all disturbances are assumed to come from Gaussian white noise. In the second analysis, technique comparisons will explicitly address the effects of quantization noise.

The investigation into various sources of error will be analyzed by first determining which parameter estimation technique produces better parameters for the improved model with no compensation for quantization disturbances (section 4.1). Next, the process will address the issue of finding the best parameter estimation technique that adequately compensates for the effects of quantization noise (section 4.2). Finally, the process will examine the effects of decimation and bias of the transformer data in order to reduce errors in the parameter estimates attributed to post processing (section 4.3).

### *Parameter estimation techniques*

The various parameter estimation techniques used in this investigation were selected from a reasonably wide range of available categories. The investigation is not exhaustive due to the fact that only five techniques were selected: least-squares (LS) estimation, prediction error (PE) estimation, output error (OE) estimation, the optimal instrumental variables (IV4) technique, and maximum likelihood (ML) estimation. These five techniques are representative, however, of the fundamental breadth of estimation techniques, that is, at least four categories of parameter estimation techniques are covered in this investigation. Least-squares (LS) and prediction error (PE) represent standard linear techniques while output error (OE) estimation represents a standard nonlinear technique. The optimal instrumental variable (IV4) technique can be either a linear or nonlinear approach depending on the choice of instruments. It was included because of its flexibility in selecting appropriate instruments. Finally, the maximum likelihood (ML) technique was selected because of its purely stochastic approach.

### *Criteria for selecting the best estimation technique*

Five criteria will be used to select the best parameter estimation technique. The estimation technique which best satisfies the majority of criteria in its implementation will be designated the best overall technique among selected techniques. Explanation of each criteria will follow.

#### 1. Does the technique provide an adequate prediction?

In order for the on-line monitoring system to detect failures in the transformer, the model must accurately predict transformer behavior. For obvious reasons, this criterion is the most important.

#### 2. Is the technique suitable for the detection of failures in the on-line monitoring system?

A parameter estimation technique will be considered unsuitable if it renders residual errors from the on-line monitoring system useless. Significant deviations provide information on possible transformer failures. If an estimation technique renders this residual information useless, the monitoring system will not detect transformer failures.

#### 3. Does the technique realize sensible physical parameters?

This criterion is extremely important in that it gives physical insight into the transformer's behavior. Physical parameters that are out of range give a good indication of an unsuitable parameter estimation technique. Acceptable physical parameters must fall in the following ranges:

$$\begin{aligned} T_o &: 60 - 600 \quad \text{minutes} \\ \theta_{ft} &: 35 - 65 \quad ^\circ\text{C} \\ R &: 2 - 10 \quad \text{dimensionless} . \end{aligned} \tag{7}$$

#### 4. Does the technique provide for the consistent estimation of parameters?

Several sets of consecutive data will be used to evaluate the estimation techniques. For example, eight days of transformer data will be divided to yield four sets of two-day length data. If the physical parameters from an estimation technique vary widely from one data set to the next consecutive data set, the parameter estimation technique will be deemed inconsistent.

#### 5. Does the technique produce a consistently minimal error covariance?

The error covariance used to validate estimated parameters in the pilot program [3] was also used in this investigation. The *valid* measure

$$\mathbf{E} = \mathbf{Y} - \Phi \hat{\theta} \tag{8}$$

$$\mathbf{C} = \frac{\mathbf{E}^T \mathbf{E}}{k + 1 - m} \left( (\Phi^T \Phi)^{-1} \right) \tag{9}$$

$$\text{valid} = \text{trace} (\mathbf{C}) \tag{10}$$

is concise in that it reflects the accuracy of all the parameters. In (8),  $\Phi$  is the regressor matrix with  $\mathbf{E}$  and  $\mathbf{Y}$  representing the error and observation vectors, respectively. The term ' $k + 1 - m$ ' is simply the length of the error vector. For good parameter estimates, the error covariance should be small.

## 4.1 Technique Comparisons with No Compensation for Quantization Effects

This section determines which parameter estimation technique produces better parameters for the improved model with no compensation for quantization disturbances, that is, the noise is not assumed to come from quantization noise. Several weeks of transformer data from a large transformer in service were utilized in this paper; however, only two-day length consecutive data sets were used in this first analysis. The selection of two-day length data sets was arbitrarily established. All five parameter estimation techniques are compared in this analysis. The goal will be to determine the technique that best meets the five selection criteria. Analysis of each technique follows.

### Least-Squares (LS) Estimated Model

Using the form of the linear regression model (11), a linear least-squares (LS) model was found to produce effectively zero mean residuals with a .36 average residual variance using various sets of transformer data. Equation (11) corresponds to equation (5) by allowing  $y$  to be  $\theta_o$  and by allowing  $\theta$  to be the vector of unknown parameters  $(K_1, K_2, K_3)$ . To complete the remainder of the substitution, the regressor vector,  $\phi$ , corresponds to a regressor matrix having three columns where the first column is assigned the values obtained from  $\theta_{top}[k-1] - \theta_{amb}[k]$ , the second column has the values of  $I^2[k]$ , and the third column is a vector of ones. Similar substitutions were performed for all the parameter estimation techniques analyzed in this paper.

$$y(t) = \phi^T(t)\theta + e(t) \quad (11)$$

The LS loss function (12) is the standard  $L_2$ -norm measure of error used to best fit the model outputs to the observed outputs. An error covariance of  $7.6 \times 10^{-3}$  was obtained for the LS approximation and will serve as a baseline for comparison since it was used in the previous study [1]. All LS parameters were estimated using the LS procedure in (13). Figure 2(a) shows the calculated and measured top oil temperature outputs.

$$\min_{\theta} \sum |y_i - \phi_i^T \theta|^2 \quad (12)$$

$$\hat{\theta} = (\Phi^T \Phi)^{-1} \Phi^T Y \quad (13)$$

When used to predict the future behavior of the transformer, the LS estimates produced significant error in both the mean and variance of the residuals (figure 2(b)). The LS technique also did not produce overall consistency of physical parameters, especially with respect to  $\theta_{fl}$  (see table 1).

### Prediction Error (PE) Estimated Model

Since prediction error (PE) is a LS technique, most LS results described earlier apply. The difference between the two rests in implementation. A PE model of the form

$$\hat{y}_k = \hat{a} y_{k-1} + \hat{b} u_{k-1} \quad (14)$$

has the advantage of adjusting a model to obtain a better fit [6]. The adjustment is caused by the PE loss function

$$\min_{\theta} E \left\{ \sum_{k=1}^{N-\tau} (\hat{y}_{k+\tau|k}(\theta) - y_{k+\tau})^2 \right\} \quad (15)$$

which obtains a prediction based on present data in order to minimize the variance of the prediction  $\tau$  steps ahead of the output. This, in essence, results in significantly minimized residuals. This advantage, however, is in effect a disadvantage to the purpose of the on-line monitoring system. Since the on-line monitoring system detects possible transformer failures by way of residual and parameter trends, the PE technique fails in implementation by drastically reducing residuals to such a point as to render information from a residual sequence useless. Figures 3(a) and 3(b) show the calculated and measured top oil temperature outputs using the PE technique. Numerical values from this estimation technique may be found in table 1.

#### Output Error (OE) Estimated Model

An output error (OE) estimated model of the form in (16) was found which produced consistent parameters among many data sets. The OE estimated model produced zero mean residuals with a residual variance of .27 on average.

$$\hat{y}_k = \hat{a} \hat{y}_{k-1} + \hat{b} u_{k-1} \quad (16)$$

This represents approximately a 25% improvement over the LS residual variance. The OE loss function has the form

$$\min_{B,F} \sum_{k=0}^{N-1} \left| y_k - \frac{B(z^{-1})}{F(z^{-1})} u_k \right|^2 \quad (17)$$

which exerts its efforts on identifying the parameters of the polynomials  $F$  and  $B$ . This polynomial notation is adopted from the standard Box-Jenkins transfer function model where  $F$  and  $B$  are polynomials in  $z^{-1}$  [7]. The error covariance of the OE approximation was found to be  $5.6 \times 10^5$  which is a two-order of magnitude improvement over the LS statistic. With the advantage of filtering the input/output data with a whitening filter, the OE technique realized sensible physical parameters over several data sets. Figure 4(a) displays the OE predicted versus the transformer's measured response. Figure 4(b) shows an OE improvement over the LS estimates when predicting the transformer's measured response two days ahead. Results from the OE technique may be viewed in table 1.

#### Optimal Instrumental Variables (IV4) Estimated Model

Since LS solutions often contain bias due to the correlation between the regressors and the prediction error, instrumental variable methods (IV4) are sometimes used. These methods generally replace the regressor  $\Phi$  used in the linear regression equation (8) with some other variable  $Z$  called the instrumental variable. Three conditions placed on  $Z$  are: 1)  $Z$  should be uncorrelated with the disturbances, 2) the matrix  $Z^T \Phi$  should be invertible, and 3)  $Z^T \Phi$  should be large in order to provide for an efficient parameter estimate. With the conditions met, the estimate takes the following form:

$$\hat{\theta} = (Z^T \Phi)^{-1} Z^T Y \quad (18)$$

The IV4 loss function is computed as the LS loss function (12) except for the use of  $Z$  in place of  $\Phi$ . The instruments selected in this analysis were developed using a four-step iterative scheme. The first step involved obtaining the LS estimate (13). The second step involved the formation of the instruments starting with (5) and computing the parameter estimates using (18). The third step involved obtaining the error between the model and the actual transformer data and using the error to estimate a first-order noise model. Finally, the last step involved calculating new instruments using the parameters obtained from step 2 and the noise filter obtained from step three. Using  $1/C(z^1)$  as a filter, the new instruments in step 4 were calculated by filtering the instruments obtained in step 2 as follows:

$$\mathbf{Z}^F[k] = \frac{1}{C(z^{-1})} \mathbf{Z}[k] \quad (19)$$

Note that all the variables in (18) were filtered in like manner. The final IV4 parameter estimates were computed using filtered variations of the variables used in (18) as follows:

$$\hat{\theta}_z = \left( \mathbf{Z}^{FT} \Phi^F \right)^{-1} \mathbf{Z}^{FT} \mathbf{Y}^F \quad (20)$$

Using the four-step procedure, an IV4 estimated model was found which produced consistent parameters among several data sets. The IV4 estimated model produced zero mean residuals with a residual variance of .30 on average. The error covariance of the IV4 approximation was found to be  $6.4 \times 10^3$  which is comparable to the LS baseline with negligible improvement. The predicted and measured top oil temperature outputs using the IV4 technique (see figures 6(a) and 6(b)) was very similar to the LS outputs. Results from the IV4 technique may be viewed in table 1.

#### Maximum Likelihood (ML) Estimated Model

Finally, maximum likelihood (ML) estimation was employed to estimate parameters for the transformer model. Using the assumption that all disturbances were normally distributed zero-mean,  $\sigma^2$  variance residuals, the ML technique obtained parameter estimates by starting with (8) to form the likelihood function, i.e.,

$$L(\theta; \mathbf{Y}) = \left( \frac{1}{\sqrt{2\pi\sigma^2}} \right)^n \exp \left\{ \frac{-1}{2\sigma^2} (\mathbf{Y} - \theta\Phi)^T (\mathbf{Y} - \theta\Phi) \right\} \quad (21)$$

Next, the log likelihood was obtained by taking the logarithm of (21) yielding

$$l(\theta; \mathbf{Y}) = \log L(\theta; \mathbf{Y}) = -\frac{n}{2} \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} (\mathbf{Y}^T \mathbf{Y} - 2\mathbf{Y}^T \theta\Phi + \Phi^T \theta^T \theta\Phi) \quad (22)$$

By taking the derivative of the log likelihood equation (22) with respect to the parameter  $\theta$ , we achieve the ML estimate

$$\hat{\theta} = (\Phi^T \Phi)^{-1} \Phi^T \mathbf{Y} \quad (23)$$

which is precisely the same as the LS estimate found in (13).

For this reason, the ML parameter estimates and results were very close to those obtained using the LS technique. Similarly, the predicted and measured top oil temperature outputs using the ML technique (see figures 8(b) and 8(c)) were very similar to the LS outputs displayed in Figure 2, that is, significant error may be seen in both the mean and variance of the residuals. The ML technique also did not produce overall consistency of physical parameters, especially with respect to  $\theta_{jt}$ . Results from the ML technique are displayed in table 1.

#### **Results**

Of the five parameter estimation techniques employed in this analysis (LS, PE, OE, IV4, and ML), the OE technique exhibited better results when the noise is assumed to be normally distributed zero-mean,  $\sigma^2$  variance residuals. The OE technique achieved this by obtaining better predictions of transformer behavior, by revealing a much improved error covariance, by achieving consistent parameter estimates, and by providing for valid and sensible physical parameters. The second best estimate was obtained using the IV4 technique due to its slightly improved error covariance (as compared to

the LS baseline) and its production of consistent parameter estimates. The LS and ML techniques tied for third place while the PE technique was considered last in this analysis due to its implementation.

## 4.2 Technique Comparisons with Compensation for Quantization Noise

As described earlier, the inputs and outputs of the on-line monitoring system are corrupted by quantization noise (figure 1(a)). This corruption corresponds to a 1°C measurement resolution which introduces 5 - 20 % error in the 5 - 20°C weekly temperature variation. It has not been determined whether or not this quantization disturbance significantly affects the estimation of parameters. To determine if the 1°C measurement resolution introduces a deleterious effect to the estimation process, this section will compare various parameter estimation techniques while analyzing whether or not the addition of a quantization term to the transformer model improves the model's prediction. It should be noted that an exhaustive quantization error analysis will not be performed. This section will only focus on one approach to compensate for the effects of the quantization disturbance. The use of other techniques to undo the effects of quantization remains a subject of future research. As described earlier, several weeks of transformer data from a large transformer in service were utilized in this paper. One-week length consecutive data sets were used in this analysis as opposed to the two-day length data sets used in section 4.1. This larger one-week length data was utilized in order to invoke reasonable statistical inferences about the quantization noise required by asymptotic theory.

Description and analysis of the additional quantization noise term, the parameter estimation techniques selected, and the results from comparing the selected techniques follow. The goal of this section is identical to that of the previous section, that is, which selected estimation technique best meets the five selection criteria described previously.

### *Quantization Noise (QN) Compensation Term*

As discussed in [1], the data from each transformer was sampled at five minute intervals with a 1°C integer value resolution. This measurement resolution introduces 5 - 20 % error in the 5 - 20°C weekly temperature variation. This may introduce significant errors in the estimation of parameters. For this reason, a statistical approach will be used to quantify the quantization effects due to the nonlinear nature of the quantizer on the input signals.

Quantization is a nonlinear and noninvertible process by which, in the case of the on-line monitoring system, the input and output signals of the transformer have been discretized into a finite number of digits, thereby producing the sampled data signals used in this paper. For rounding quantizers, each sample is rounded and assigned to the nearest level. Quantization error is often defined to be the difference between the original sampled inputs and the rounded inputs. It is reasonable to assume that the probability distribution of the error process is uniform over the range of the quantization error [9], that is, the amplitude of the quantization noise is in the range from -.5°C to +.5°C given a 1°C integer value resolution (figure 8(a)). In this paper, we assume that the successive noise samples are uncorrelated with each other and that the quantization noise is uncorrelated with the input signals, thus the quantization noise is assumed to be a uniformly distributed white noise sequence with a mean value of 0 and a variance of 1/12.

For the development of the quantization term, we will commence from the actual transformer signals to the sampled data and eventually to its additive effect in the improved model. Since the sampled data will be different than the original transformer data, let us denote the quantization error to be the difference between the quantized sample data (denoted  $\theta_{amb}$ ,  $I$ , and  $\theta_{top}$ ) and the true data (denoted  $\theta_{amb}^{TRUE}$ ,  $I^{TRUE}$ , and  $\theta_{top}^{TRUE}$ ), that is,

$$e_q[k] = \theta_{amb}^{TRUE} - \theta_{amb} \quad (24)$$

or

$$e_q[k] = I^{TRUE} - I \quad (25)$$

or

$$e_q[k] = \theta_{top}^{TRUE} - \theta_{top} \quad (26)$$

Starting with the improved model (5), we will trace the development of a quantization disturbance term,  $q_n[k]$ , by replacing the sampled data with the true data as follows:

$$\theta_{top}[k] = K_1 \theta_{top}[k-1] + (1-K_1) \theta_{amb}[k] + K_2 I[k]^2 + K_3 \quad (27)$$

$$\theta_{top}^{TRUE}[k] - e_q[k] = K_1 (\theta_{top}^{TRUE}[k-1] - e_q[k-1]) + (1-K_1) (\theta_{amb}^{TRUE}[k] - e_q[k]) + K_2 (I^{TRUE}[k] - e_q[k])^2 + K_3 \quad (28)$$

$$\theta_{top}^{TRUE}[k] = K_1 \theta_{top}^{TRUE}[k-1] + (1-K_1) \theta_{amb}^{TRUE}[k] + K_2 I^{TRUE}[k]^2 + K_3 + q_n[k] \quad (29)$$

where

$$q_n[k] = K_1 (e_q[k] - e_q[k-1]) - 2K_2 I^{TRUE}[k] e_q[k] + K_2 e_q[k]^2 \quad (30)$$

With the addition of quantization noise sequences to the collected data, namely  $\theta_{amb}$ ,  $I$ , and  $\theta_{top}$ , the improved model may now be written in a form that describes both the true data from the transformer and the additive effects of a quantization disturbance term. This, in effect, is equivalent to modeling the transformer and the sensors used to acquire the time-varying data. The term  $e_q[k]$  in equations (28), (29) and (30) represents the quantization errors caused by the rounding of the collected data. The disturbance term  $q_n[k]$  is the incorporation of all quantization errors of the model. As seen in (30),  $q_n[k]$  is a nonlinear function of the quantization errors  $e_q[k]$  and the true load current  $I^{TRUE}[k]$ . Since the true data,  $(\theta_{amb}^{TRUE}, I^{TRUE}, \text{ and } \theta_{top}^{TRUE})$ , are irretrievable, an exact formulation of the transformer model is impossible. This also implies that some errors will exist when estimating parameters based on the improved model and the given corrupted data. Given the nature of quantization noise, it is reasonable to obtain results using the stochastic approach of assuming a random noise sequence for the quantization disturbance term.

### ***Parameter Estimation Techniques Selected***

The parameter estimation techniques selected for this analysis differs slightly from the ones selected in section 4.1. The LS technique was not included explicitly due to the fact that the results were very close to those using the ML technique (as discussed in section 4.1). The PE technique was not utilized because it failed in its implementation, that is, it drastically reduced residuals to such a point as to render information from the residual sequence useless. This point was explained in section 4.1. Two OE techniques were employed. The first OE technique uses the improved model with no additional quantization disturbance term. The second OE technique, OE (qn), technique uses the improved model with the additional quantization disturbance sequence. And in like manner, two IV4 techniques were employed. The first IV4 technique uses the improved model with no additional quantization term while the second technique, IV4 (qn), uses the improved model with the additional quantization disturbance sequence. Finally, the ML technique will be utilized in the same manner as before. The list of selected parameter estimation techniques analyzed in this section follows:

- OE
- OE (qn)
- IV4
- IV4 (qn)
- ML

For notational purposes, the two parameter estimation techniques with the notation (qn) represent techniques applied with the quantization disturbance term added to the improved model.

### Output Error (OE) Estimated Model

An output error (OE) technique was applied to MIT's improved model to estimate the various parameters in (5). The OE estimated model produced zero mean residuals with a .35 residual variance on average. The error covariance of the OE approximation was  $2.4 \times 10^{-5}$  for one week of data with similar results for remaining weeks. Consistent and sensible physical parameters were realized with this technique. Figure 4(c) displays the OE predicted versus measured response for week 2 in the transformer data set. This technique also provided good one- and two-week predictions of transformer behavior, as seen in figures 4(d) and 4(e), respectively. Results from the OE technique including prediction statistics are displayed in table 2.

### Output Error (OE) Estimated Model with additional (qn) term

The OE (qn) technique produced results that were virtually identical to those using the OE technique. The error covariance was identical and the physical parameters ( $T_o, \theta_{ff}, R$ ) were comparable and sensible. The fit and weekly predictions of the OE (qn) estimated model (figures 5(a)-5(c)) were practically identical to the OE responses. Results are shown in table 2.

### Optimal Instrumental Variables (IV4) Estimated Model

An optimal instrumental variables (IV4) technique was applied to the improved model. Results from this technique revealed zero mean residuals with a residual variance of .36 on average. All estimated physical parameters were found to be consistent and sensible among several weeks of transformer data. The IV4 estimated model differs from the OE estimated model with respect to its error covariance. The IV4 error covariance was not very good at  $3.9 \times 10^3$  which is a two-order of magnitude decrease in parameter reliability. The IV4 fit is shown in figure 6(c). The one- and two-week predictions (figures 6(d) and 6(e)) were comparable to those obtained using the OE technique.

### Optimal Instrumental Variables (IV4) Estimated Model with additional (qn) term

The IV4 (qn) technique nearly produced zero mean residuals with a .34 residual variance on average. The IV4 (qn) method produced the worst results of all compared techniques in this section. First, the estimated physical parameters were not sensible particularly with respect to  $\theta_{ff}$  and  $R$ . Second, the estimated parameters were not found to be consistent. There were large fluctuations in the values for  $\theta_{ff}$  (see table 2). Third, the estimated parameters were not found to be as valid as those of the OE technique. The  $3.2 \times 10^{-2}$  error covariance was at least three orders of magnitude below that of the OE technique. The IV4 (qn) fit differed slightly from the IV4 estimated model (figure 7(a)). Unacceptable one- and two-week predictions for the IV4 (qn) technique are shown in figures 7(b) and 7(c), respectively.

### Maximum Likelihood (ML) Estimated Model

Maximum likelihood (ML) estimation was applied to the improved model. In the quantization noise discussion, it was generally believed that the probability distribution of the error process was uniform over the range of the quantization error. It is assumed that the successive noise samples are uncorrelated with each other and that the quantization noise is uncorrelated with the input signals. Given the use of large sample sizes (one-week length data sets), the ML technique employed the use of an approximating (gaussian) distribution to characterize the stochastic behavior of the quantization noise sequences. This implication is based on the central limit theorem due to the fact that the sample size is large and each quantization sequence in (30) has the same uniform distribution.

The ML estimated model nearly produced zero mean residuals with a .49 residual variance on average. The estimated physical parameters were not sensible particularly with respect to  $\theta_{ff}$  and  $R$ . With an error covariance of  $2.8 \times 10^{-3}$ , the estimated parameters were not as reliable as the OE estimates. The most positive aspect of the ML estimated model was the fairly consistent estimation of physical parameters. The ML fit is shown in figure 8(d). The one- and two-week



predictions (figures 8(e) and 8(f)) were not as reliable as the OE predictions but were much better than the IV4 (qn) predictions.

### Other Models

Several other parameter estimation techniques (and modified versions of the techniques listed above) were analyzed in this section other than the ones described. An independent OE technique (i.e., the terms  $K_i$  and  $(1-K_i)$  in (5) were estimated independently) was estimated, but the results (not shown) were not consistent. Another optimal instrument variables (IV4) technique was employed using a 2<sup>nd</sup> order noise model. Results obtained were identical to the IV4 method described earlier in this section. A state-space model [8] was identified but was found to be unsuitable for the on-line monitoring system for the same reasoning as for the prediction error (PE) technique discussed in section 4.1. Finally, a Lesieutre algorithm (LA) was used to estimate parameters but did not obtain good estimates. This was due in part to the rank deficiency found in the very large (sometimes 500 x 500 dimension) regression matrix.

### **Results**

The additional quantization noise term did not improve the estimation of parameters for the improved model regardless of the parameter estimation technique selected. In the case of the IV4 estimated model, the additional term in IV4 (qn), decreased the reliability of the parameter estimates. This same statement cannot be said with regard to the OE (qn) model. The additional quantization term, though not decreasing the validity of the parameter estimates, did not increase it either since the results from both OE and OE (qn) techniques were practically identical. Given the similar results, it is advisable to utilize the simplest model, that is, the OE estimated model.

Of the five parameter estimation techniques employed in this analysis, the OE technique should be used primarily on MIT's improved transformer top oil temperature rise over ambient temperature model when using long data sets (a week or greater). Implementation of the OE technique exhibited better results than all of the described estimation techniques in the presence of quantization noise. The OE technique obtained accurate one- and two-week predictions of transformer behavior, revealed the best error covariance found among the estimation techniques, obtained consistent parameter estimates, and provided for valid and sensible physical parameters. The second best estimate was obtained using the OE (qn) technique due to its similar OE results. The IV4 technique was the third best while the ML technique came in fourth place. The IV4 (qn) technique was considered last in this analysis due to its inadequate prediction and its unacceptable error covariance.

## **4.3 Decimation and Bias**

This section will examine the effects of decimation (missing data) and bias of the transformer data in order to reduce errors in the parameter estimates attributed to post processing. As described in [1], a third possible source of error in the transformer model may be the five minute sampling period originally selected to enable the consistent estimation of parameters in MIT's pilot transformer facility. In order to reduce errors in the parameter estimates attributed to post processing, two approaches will be analyzed. The first will examine the effects of down sampling (decimation) for the improved model. The second approach will address an unanswered question which surfaced when dealing with the effects of quantization noise, that is, will biasing the data adequately compensate for the quantization noise.

### ***Decimation***

The implementation of two estimation techniques (the best technique, OE, and the worst technique, PE, from section 4.1) were compared using the same transformer data (see table 3). At least four observations will be discussed with respect to down sampling of the data. First, the thermal time constant,  $T_o$ , was observed to be the most sensitive physical parameter to down sampling among many data sets and estimation techniques. Second, the degree of error introduced into the parameter estimates by decimation is directly proportional, in a general sense, to the estimation technique employed. The OE technique showed less than 1% error in the thermal time constant at a decimation factor of 10. The

PE technique, on the other hand, introduced 67 % error into  $T_o$  at the same decimation factor. The data suggests that an OE technique should be used in order to minimize the errors in the estimated parameters attributed to decimation.

The third observation in this analysis revealed that the decimation factor had negligible effect on residual variances,  $\sigma_e^2$ . This is a positive result for the on-line monitoring system since transformer failures are detected by checking for deviations in the residual sequence. Constant residual variances (unaltered by the decimation factor) can be used as statistical bounds on the residual error. This allows the on-line monitoring system to have greater confidence in detecting transformer failures independent of the decimation factor used during post processing.

The last observation concerns the error covariance, *valid*, described in (10). Parameter reliability is maximized when decimation is not implemented (see table 3). This is an obvious conclusion due to the fact that all of the information contained in the data is utilized in the estimation process. What is commonly not known is the extent to which decimation introduces errors in the parameter estimates. Regardless of the parameter estimation technique used, the error covariance increased at most by two orders of magnitude using any decimation factor greater or equal to five. This signifies the extent to which decimation lessens parameter reliability.

### ***Bias***

Biassing the data by discrete and floating-point values was thought to be an adequate compensation to cancel the effects of quantization noise. This, however, did not turn out to be the case. Biassing the data before applying a parameter estimation technique was found to have no effect on the physical parameters or the residual statistics.

## **5 Conclusions and Recommendations**

Various sources of errors in MIT's improved model and estimation process have been examined in this paper. Possible sources of errors were the estimation of parameters, the quantization noise found in the collected data, and the effects of post-processing (data sampling and constant biases). Results from this paper show that errors were indeed produced by all three sources. An investigation into the extent and possible remedy of the error sources was conducted. Results from this paper show that an output error (OE) parameter estimation technique should be selected to replace the current least squares estimation technique. When compared to results from several other parameter estimation techniques, the output error technique obtained accurate predictions of transformer behavior, revealed the best error covariance, obtained consistent parameter estimates, and provided for valid and sensible using two-day length data sets. The OE technique also proved best (among selected techniques) in the presence of quantization noise using one-week length data sets by satisfying the selection criteria. For these reasons, the OE technique is designated the best overall technique among selected techniques by best satisfying all of the selection criteria in its implementation. The primary recommendation in this paper is the selection of an OE technique to replace the current least squares (LS) estimation technique.

Decimation of the transformer data also introduced significant error into the parameter estimates, thereby affecting parameter reliability. With regard to decreased validity of parameters, the error covariance increased at most by two orders of magnitude using any decimation factor greater or equal to five. This confirms the notion that the greater the decimation, the less the parameter reliability. To minimize the introduction of errors into the parameter estimates, an appropriate parameter estimation technique should be selected. This paper recommends the use of the OE technique to minimize errors attributed to decimation of the transformer data. To obtain greater validity of the parameter estimates, the decimation factor should be lowered. As a positive note, decimation had negligible effect on residual variances.

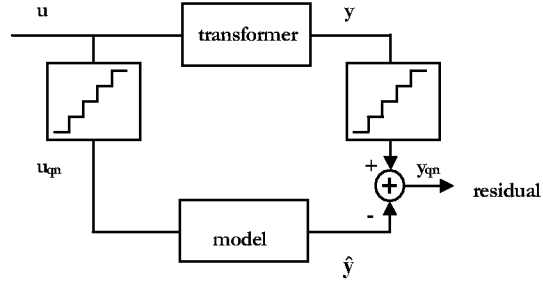
Further research is required to address the use of other techniques to undo the effects of quantization noise as well as the nonlinear estimation of the parameters associated with MIT's improved model.

## 6 Acknowledgments

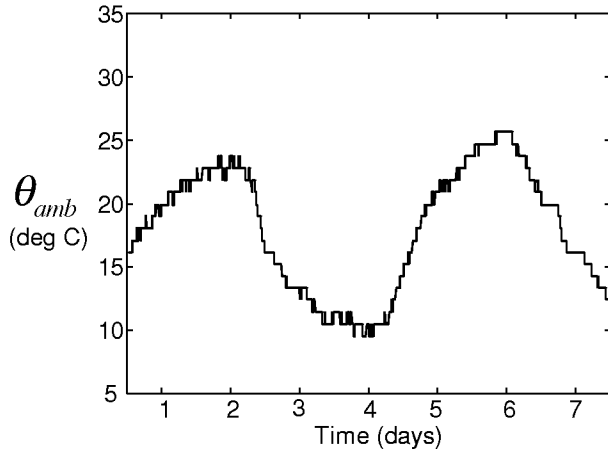
The work presented here was supported by MIT Professor Bernie C. Lesieutre. Data used in this report was obtained from a large power transformer in service. This paper describes system identification research performed at the Massachusetts Institute of Technology and supported by both the National Space Club and the National Aeronautics and Space Administration's George M. Low Engineering Fellowship.

## References

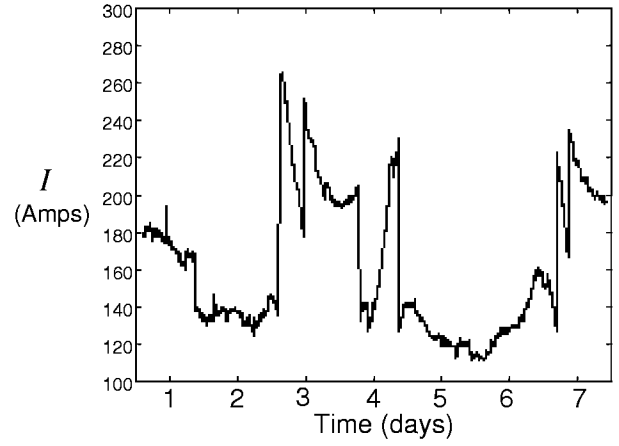
- [1] B. C. Lesieutre, W. H. Hagman, and J. L. Kirtley Jr., "An Improved Transformer Top Oil Temperature Model for Use in An On-Line Monitoring and Diagnostic System," presented at the *IEEE/PES Winter Meeting*, January 21-25, 1996.
- [2] J. L. Kirtley Jr., W. H. Hagman, B. C. Lesieutre, M. J. Boyd, E. P. Warren, H. P. Chou, R. D. Tabors, "Model-based monitoring of transformers," *IEEE Computer Applications in Power*, January, 1996.
- [3] T. H. Crowley, *Automated Diagnosis of Large Power Transformers Using Adaptive Model-Based Monitoring*, S.M. thesis, Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, Cambridge, MA, 1990.
- [4] L. E. Edwards, *An Adaptive Thermal Module for Transformer Monitoring*, S.M. thesis, Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, Cambridge, MA, 1988.
- [5] IEEE Guide for Loading Mineral-Oil-Immersed Power Transformers Rated in Excess of 100 MVA (65°C), IEEE C.57.115.1991.
- [6] R. Johansson, *System Modeling and Identification*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1993.
- [7] L. Ljung, *System Identification*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1987.
- [8] J. Juang, *Applied System Identification*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1994.
- [9] Oppenheim, A. V., and Schafer, R. W., *Discrete-Time Signal Processing*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1989.



(a) MIT's adaptive, on-line monitoring system.

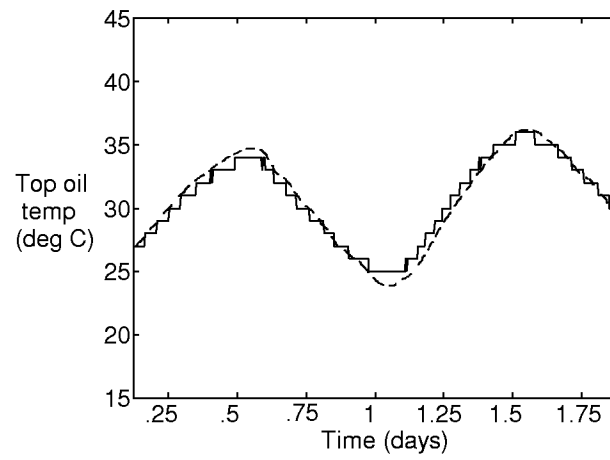


(b) Ambient temperature input signal.

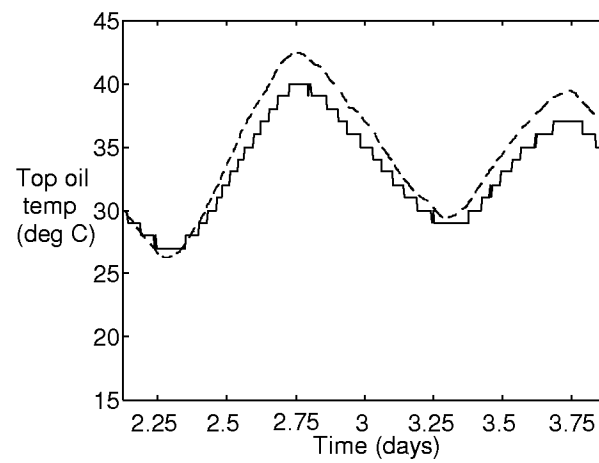


(c) Load current input signal.

Figure 1. MIT's adaptive, on-line monitoring system with input signals.

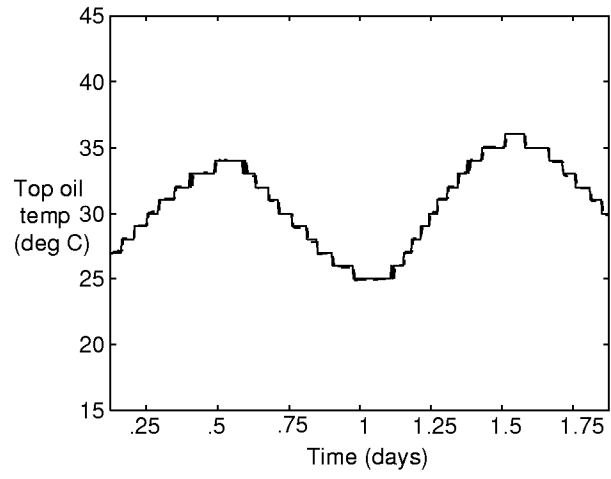


(a) Fitting model to discrete data over two days.

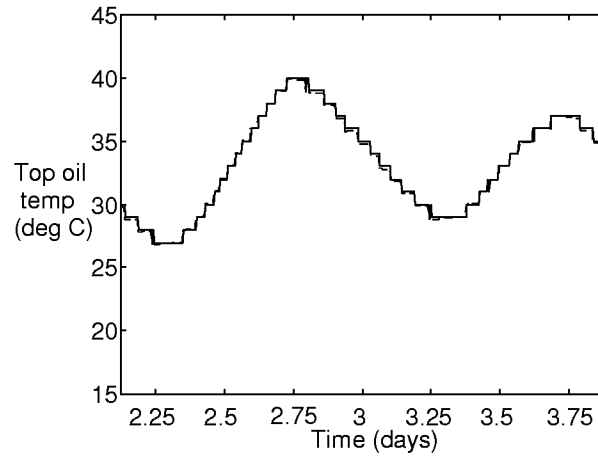


(b) Predicting ahead two days.

Figure 2. Least squares (LS) prediction of top oil temperature in transformer.

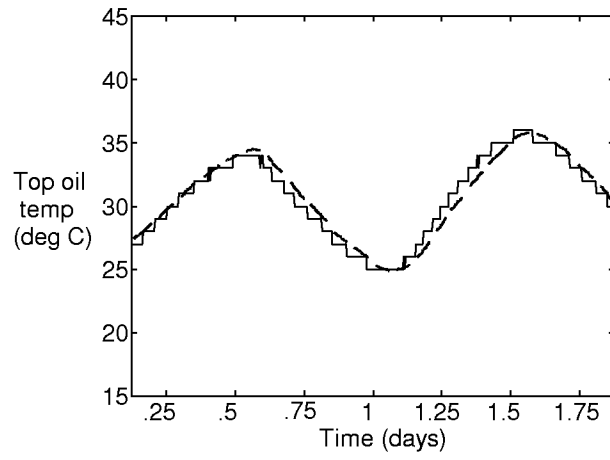


(a) Fitting model to discrete data over two days.

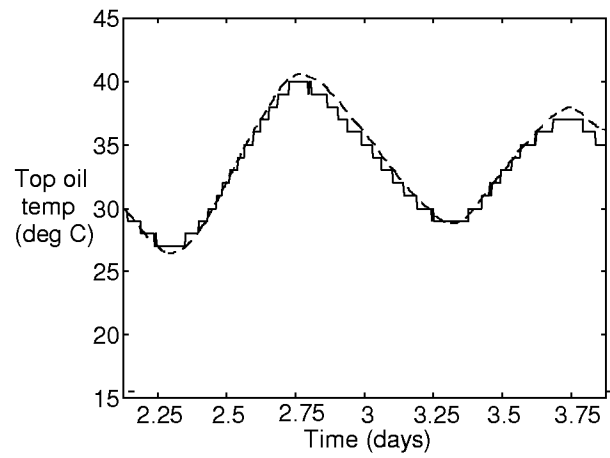


(b) Predicting ahead two days.

Figure 3. Prediction error (PE) prediction of top oil temperature in transformer.

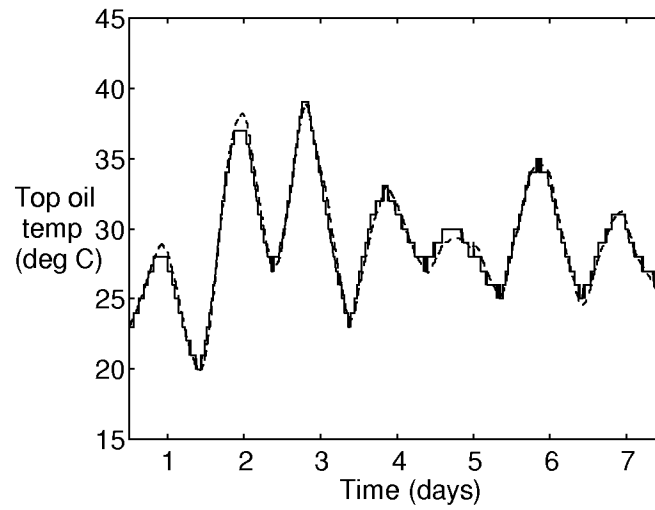


(a) Fitting model to discrete data over two days.

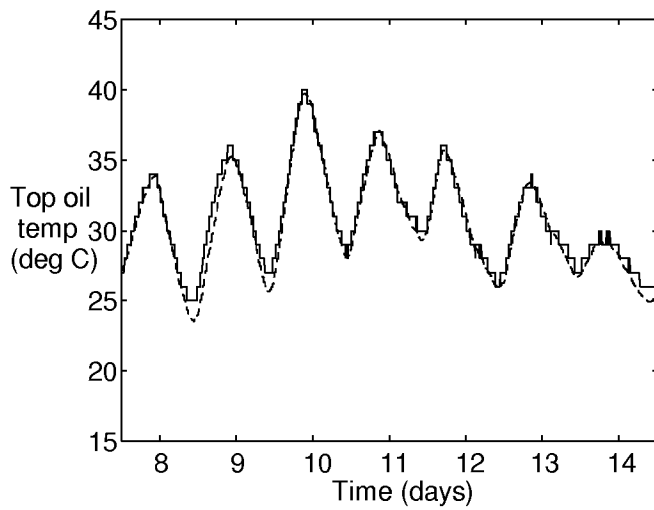


(b) Predicting ahead two days.

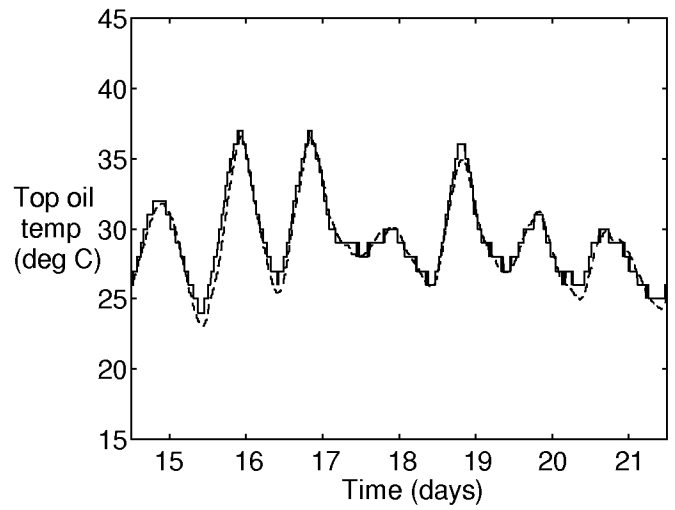
Figure 4. Output error (OE) prediction of top oil temperature in transformer.



(c) Fitting model to discrete data over one week.



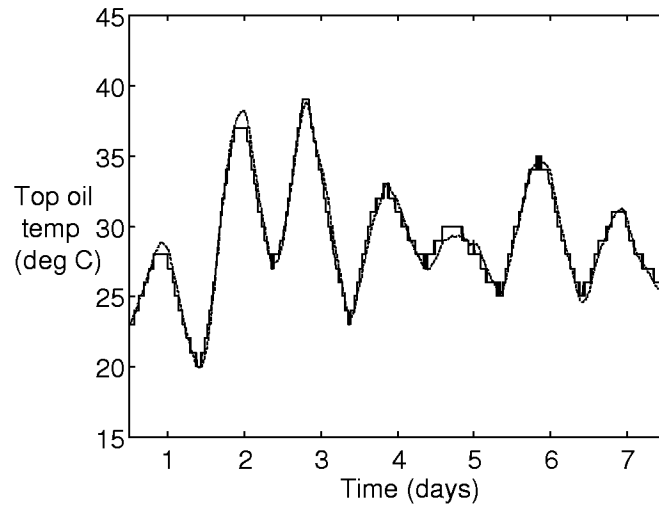
(d) Predicting ahead one week.



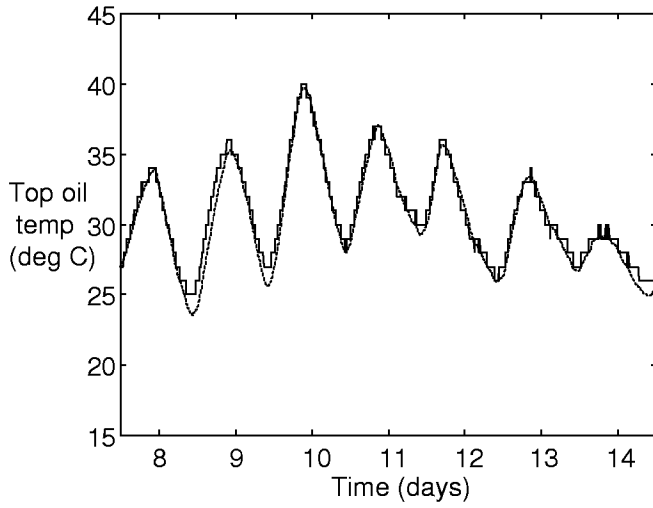
(e) Predicting ahead two weeks.

Figure 4. Continued.

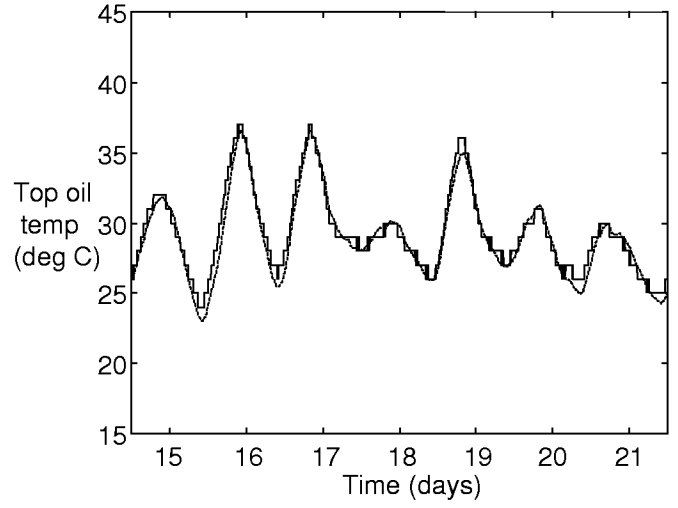




(a) Fitting model to discrete data over one week.

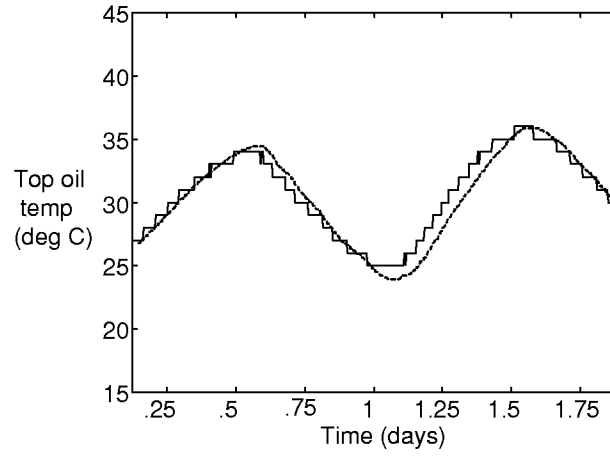


(b) Predicting ahead one week.

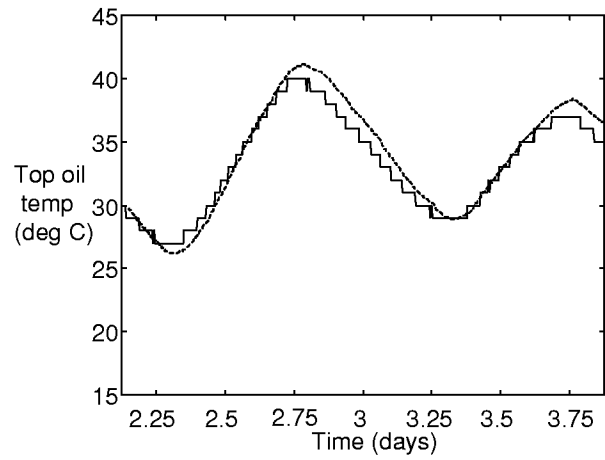


(c) Predicting ahead two weeks.

Figure 5. Output error (qn) prediction of top oil temperature in transformer.

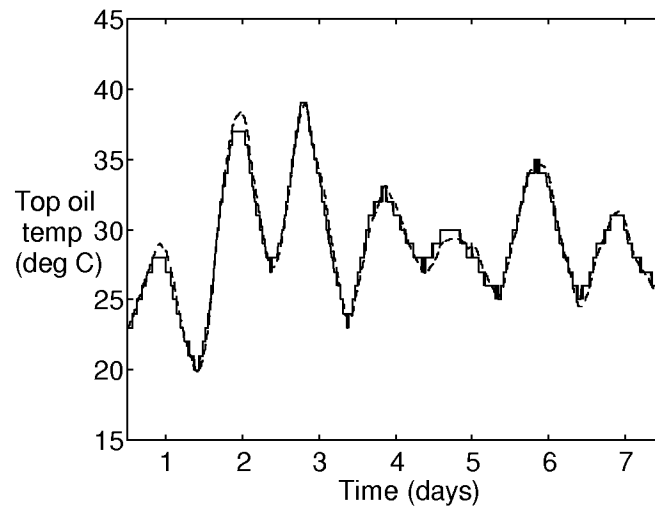


(a) Fitting model to discrete data over two days.

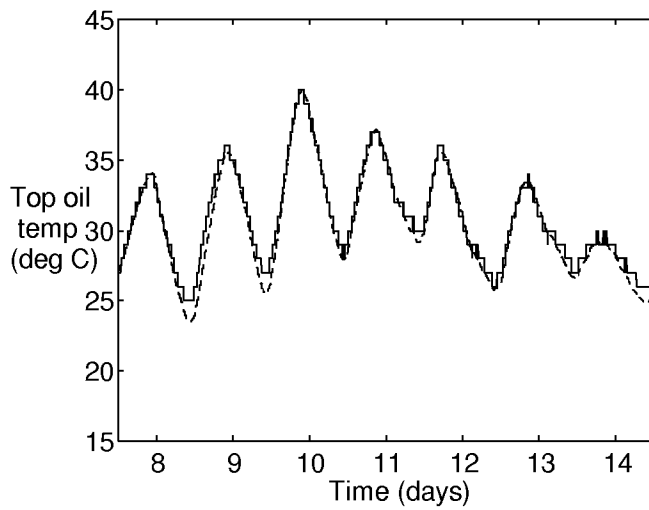


(b) Predicting ahead two days.

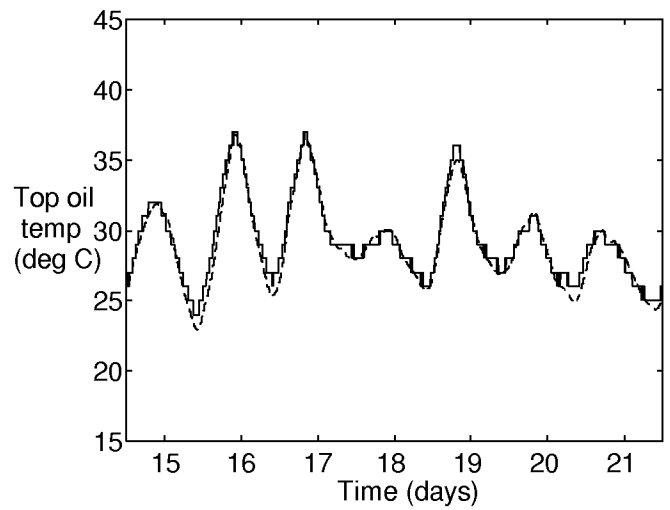
Figure 6. Optimal instrumental variables (IV4) prediction of top oil temperature in transformer.



(c) Fitting model to discrete data over one week.

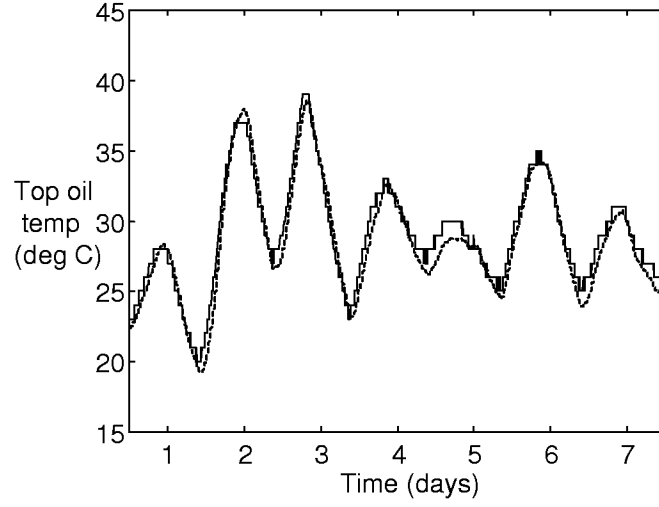


(d) Predicting ahead one week

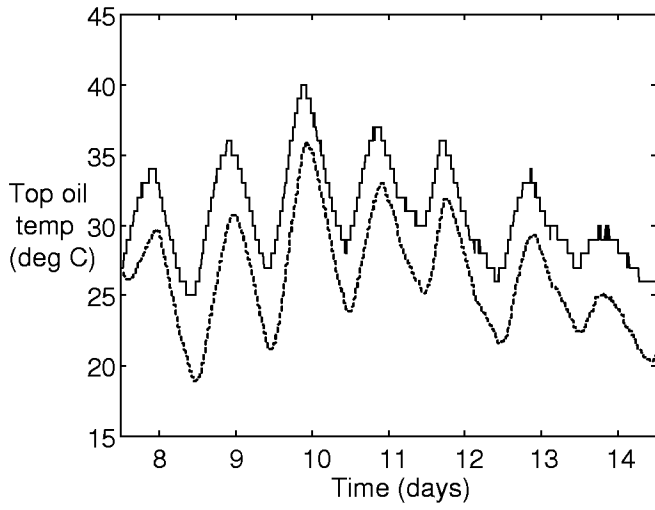


(e) Predicting ahead two weeks.

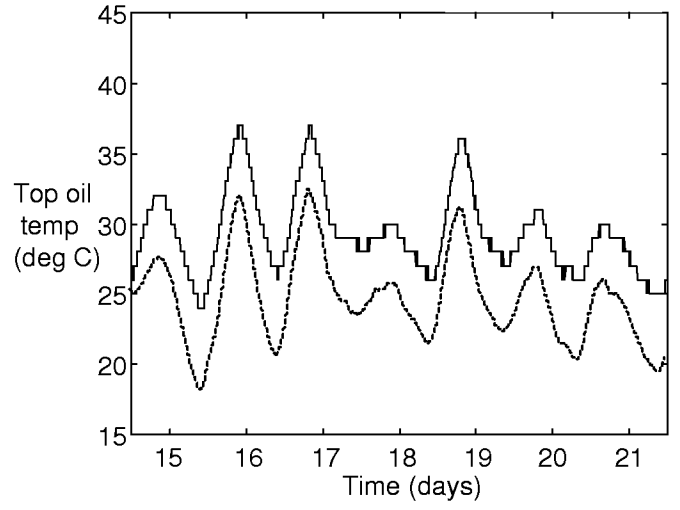
Figure 6. Continued.



(a) Fitting model to discrete data over one week.

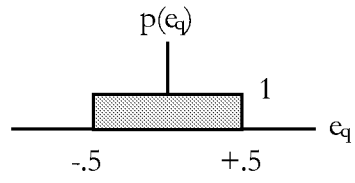


(b) Predicting ahead one week

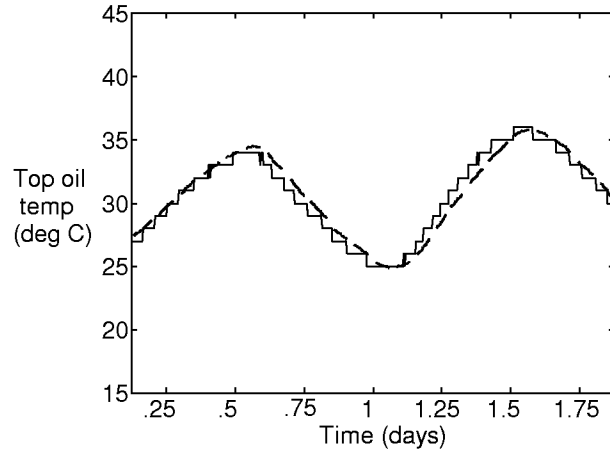


(c) Predicting ahead two weeks.

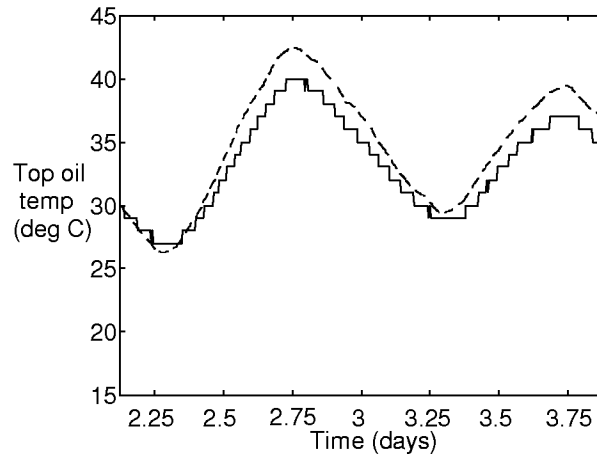
Figure 7. Optimal instrumental variables (qn) prediction of top oil temperature in transformer.



(a) Uniform probability density function of quantization error.

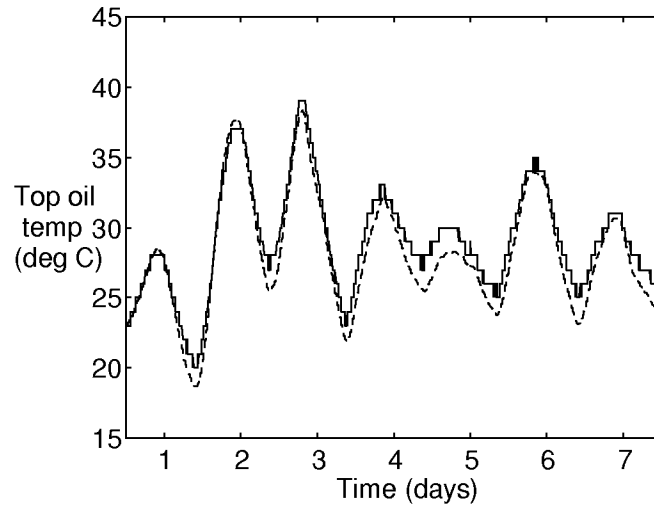


(b) Fitting model to discrete data over two days.

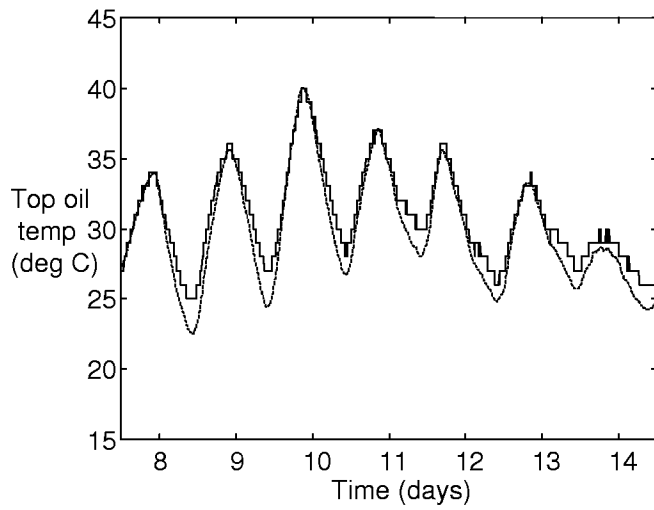


(c) Predicting ahead two days.

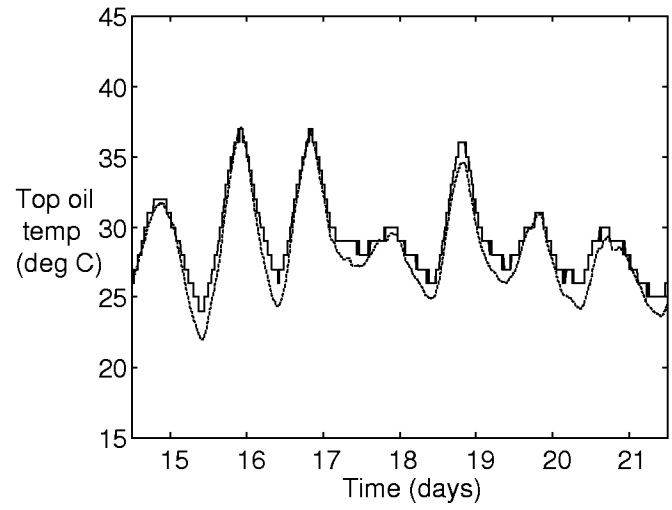
Figure 8. Maximum likelihood (ML) prediction of top oil temperature in transformer.



(d) Fitting model to discrete data over one week.



(e) Predicting ahead one week



(f) Predicting ahead two weeks.

Figure 8. Continued.

Table 1. Two-day Length Parameter Estimates and Prediction Statistics

Parameter Estimation Technique	Data	Model Fit over Two-Day Length Data						Prediction Error Statistics	
		Estimated Parameters			Errors in Fit			Next Two Days	
		$T_o$	$\theta_{fl}$	R	<i>valid</i>	$\mu_e$	$\sigma_e^2$	$\mu_e$	$\sigma_e^2$
LS	days 1-2	175	102	8.6	$7.6 \times 10^{-3}$	-.03	.44	-1.44	.94
	days 3-4	206	72	5.8	$1.8 \times 10^{-2}$	.05	.32	-.19	.26
	days 5-6	190	42	2.5	$1.6 \times 10^{-2}$	.04	.34	.05	.24
	days 7-8	132	34	1.6	$4.0 \times 10^{-2}$	.02	.34	---	---
PE	days 1-2	176	102	8.6	$7.6 \times 10^{-3}$	0	.07	.04	.08
	days 3-4	206	72	5.8	$1.8 \times 10^{-2}$	0	.08	0	.08
	days 5-6	190	42	2.5	$1.6 \times 10^{-2}$	0	.08	0	.06
	days 7-8	132	34	1.6	$4.0 \times 10^{-2}$	0	.06	---	---
OE	days 1-2	220	57	3.8	$5.6 \times 10^{-5}$	-.01	.46	-.34	.28
	days 3-4	242	13	.04	$7.6 \times 10^{-5}$	.01	.20	.17	.42
	days 5-6	230	60	4.5	$8.9 \times 10^{-5}$	0	.23	.21	.29
	days 7-8	274	42	2.5	$5.5 \times 10^{-4}$	-.05	.21	---	---
IV4	days 1-2	211	74.8	5.5	$6.4 \times 10^{-3}$	-.02	.45	-.76	.40
	days 3-4	230	71.9	5.8	$1.5 \times 10^{-2}$	.08	.28	-.28	.24
	days 5-6	225	48.6	3.1	$1.5 \times 10^{-2}$	0	.24	.07	.23
	days 7-8	267	50.8	3.4	$6.2 \times 10^{-2}$	-.02	.22	---	---
ML	days 1-2	175	102	8.7	$7.6 \times 10^{-3}$	-.03	.44	-1.44	.94
	days 3-4	206	72.3	5.8	$1.8 \times 10^{-2}$	.05	.32	-.19	.26
	days 5-6	190	42	2.5	$1.7 \times 10^{-2}$	.04	.34	.05	.24
	days 7-8	132	34.2	1.6	$4.0 \times 10^{-2}$	.02	.34	---	---

Table 2. Weekly Parameter Estimates and Prediction Statistics

Parameter Estimation Technique	Data Week	Model Fit over Week						Prediction Error Statistics			
		Estimated Parameters			Errors in Fit			Next 8-14 Days		Next 15-21 Days	
		$T_o$	$\theta_{fl}$	R	$valid$	$\mu_e$	$\sigma_e^2$	$\mu_e$	$\sigma_e^2$	$\mu_e$	$\sigma_e^2$
OE	week 1	235	43.3	2.7	$4.8 \times 10^{-5}$	0	.35	.30	.57	.50	.31
	week 2	227	55.9	4.0	$2.4 \times 10^{-5}$	0	.39	.41	.36	.32	.41
	week 3	236	39.2	2.2	$1.2 \times 10^{-5}$	0	.30	-.05	.48	---	---
	week 4	215	49.2	3.2	$2.6 \times 10^{-5}$	0	.37	---	---	---	---
OE (qn)	week 1	237	43.3	2.7	$4.8 \times 10^{-5}$	0	.36	.30	.58	.50	.31
	week 2	227	55.7	4.0	$2.4 \times 10^{-5}$	0	.39	.43	.36	.32	.42
	week 3	237	39.0	2.1	$1.2 \times 10^{-5}$	0	.31	-.05	.47	---	---
	week 4	214	49.2	3.2	$2.6 \times 10^{-5}$	0	.37	---	---	---	---
IV4	week 1	218	41.2	2.5	$7.6 \times 10^{-3}$	-.02	.34	.30	.54	.46	.32
	week 2	219	55.2	3.9	$4.2 \times 10^{-3}$	-.02	.39	.39	.36	.29	.39
	week 3	228	46.7	2.9	$2.5 \times 10^{-3}$	0	.31	-.07	.40	---	---
	week 4	213	50.9	3.3	$3.9 \times 10^{-3}$	0	.37	---	---	---	---
IV4 (qn)	week 1	217	85.8	9.6	$3.2 \times 10^{-2}$	-.01	.34	.90	1.26	1.89	.82
	week 2	220	62.8	8.3	$4.8 \times 10^{-3}$	-.03	.40	4.21	.63	4.07	.63
	week 3	235	137.3	14.1	$1.9 \times 10^{-2}$	0	.30	-1.58	3.24	---	---
	week 4	220	108.4	12.0	$1.1 \times 10^{-2}$	0	.33	---	---	---	---
ML	week 1	153	31.0	1.5	$7.0 \times 10^{-3}$	0	.56	0	.56	.39	.94
	week 2	174	47.0	3.0	$4.6 \times 10^{-3}$	-.01	.55	.27	.57	.19	.43
	week 3	195	47.9	3.0	$2.8 \times 10^{-3}$	0	.40	-.07	.37	---	---
	week 4	168	45.0	2.8	$4.3 \times 10^{-3}$	0	.45	---	---	---	---



Table 3. Decimation Effects

Estimation Technique	Parameters & Residuals	Decimation Factor		
		1	5	10
OE	$T_o$	236	236	234
	$\theta_{fl}$	41.0	39.2	39.2
	R	2.3	2.2	2.2
	<i>valid</i>	$7.2 \times 10^{-7}$	$1.2 \times 10^{-5}$	$4.4 \times 10^{-5}$
	$\sigma_e^2$	.32	.30	.29
PE	$T_o$	121	195	201
	$\theta_{fl}$	54.7	47.9	50.0
	R	3.7	3.0	3.2
	<i>valid</i>	$1.6 \times 10^{-3}$	$2.8 \times 10^{-3}$	$4.3 \times 10^{-3}$
	$\sigma_e^2$	.07	.07	.11

<b>REPORT DOCUMENTATION PAGE</b>			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
<b>1. AGENCY USE ONLY</b> (Leave blank)		<b>2. REPORT DATE</b> January 1999		<b>3. REPORT TYPE AND DATES COVERED</b> Technical Memorandum
<b>4. TITLE AND SUBTITLE</b> Comparing Parameter Estimation Techniques for an Electrical Power Transformer Oil Temperature Prediction Model			<b>5. FUNDING NUMBERS</b>  WU 519-50-51-02	
<b>6. AUTHOR(S)</b> A. Terry Morris				
<b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b>  NASA Langley Research Center Hampton, VA 23681-2199			<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b>  L-17642	
<b>9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b>  National Aeronautics and Space Administration Washington, DC 20546-0001			<b>10. SPONSORING/MONITORING AGENCY REPORT NUMBER</b>  NASA/TM-1999-208974	
<b>11. SUPPLEMENTARY NOTES</b>				
<b>12a. DISTRIBUTION/AVAILABILITY STATEMENT</b> Unclassified-Unlimited Subject Category 63      Distribution: Standard Availability: NASA CASI (301) 621-0390			<b>12b. DISTRIBUTION CODE</b>	
<b>13. ABSTRACT</b> (Maximum 200 words) This paper examines various sources of error in MIT's improved top oil temperature rise over ambient temperature model and estimation process. The sources of error are the current parameter estimation technique, quantization noise, and post-processing of the transformer data. Results from this paper will show that an output error parameter estimation technique should be selected to replace the current least squares estimation technique. The output error technique obtained accurate predictions of transformer behavior, revealed the best error covariance, obtained consistent parameter estimates, and provided for valid and sensible parameters. This paper will also show that the output error technique should be used to minimize errors attributed to post-processing (decimation) of the transformer data. Models used in this paper are validated using data from a large transformer in service.				
<b>14. SUBJECT TERMS</b> Parameter Estimation, Quantization Noise, Large Power Transformers, Output Error, Optimal Instrumental Variables, Decimation, and Adaptive Monitoring			<b>15. NUMBER OF PAGES</b> 34	
			<b>16. PRICE CODE</b> A03	
<b>17. SECURITY CLASSIFICATION OF REPORT</b> Unclassified	<b>18. SECURITY CLASSIFICATION OF THIS PAGE</b> Unclassified	<b>19. SECURITY CLASSIFICATION OF ABSTRACT</b> Unclassified	<b>20. LIMITATION OF ABSTRACT</b> UL	